9.2 #22

$$ABC = \begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -1\cdot 2 + 0\cdot -1 & -1\cdot 3 + 0\cdot 1 \\ 1\cdot 2 + 2\cdot -1 & 3\cdot 1 + 2\cdot 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -3 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -2\cdot 1 + -3\cdot 0 & -2\cdot 2 + -3\cdot -1 \\ 0\cdot 1 + 5\cdot 0 & 0\cdot 2 + 5\cdot -1 \end{pmatrix}$$

$$= \begin{pmatrix} -2 & -1 \\ 0 & -5 \end{pmatrix}$$

9.2 #28

(a) $m = 4,\ n \in \mathbb{Z}^+$

(b) $n = 3,\ m \in \mathbb{Z}^+$

9.2 #32

(a) $AB = \begin{bmatrix} 1 & 4 & -2 \end{bmatrix}\begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} = 1\cdot -1 + 2\cdot 4 + 3\cdot -2 = 1$

note that this is a 1x3 times 3x1 matrix multiplication. So we should get a 1x1 matrix as a result.

(b) $BA = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 1 & 4 & -2 \end{bmatrix} = \begin{pmatrix} -1\cdot 1 & -1\cdot 4 & -1\cdot -2 \\ 2\cdot 1 & 2\cdot 4 & 2\cdot -2 \\ 3\cdot 1 & 3\cdot 4 & 3\cdot -2 \end{pmatrix} = \begin{pmatrix} -1 & -4 & 2 \\ 2 & 8 & -4 \\ 3 & 12 & -6 \end{pmatrix}$

Similarly, a 3x1 times 1x3 multiplication should result in a 3x3 matrix.

9.2 #52

(a) given $X = AX + D$, we then have

$$X - AX = D$$

$$\Rightarrow IX - AX = D$$

$$\Rightarrow (I-A)X = D$$

$$\Rightarrow (I-A)^{-1}(I-A)X = (I-A)^{-1}D$$ note that we can do this since $I-A$ invertible.

$$\Rightarrow IX = X = (I-A)^{-1}D$$ as desired.

(b) $I - A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ 0 & 2 \end{pmatrix}$, $\det(I-A) = -2 \cdot 2 + -2 \cdot 0 = -4$

using the method from extra exercise, we can compute $(I-A)^{-1}$:

$$(I-A)^{-1} = -\frac{1}{4}\begin{pmatrix} 2 & 2 \\ 0 & -2 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$$

Finally, we compute for X:

$$X = -\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -2 \\ 2 \end{pmatrix} = -\frac{1}{2}\begin{pmatrix} 1 \cdot -2 + 1 \cdot 2 \\ 0 \cdot -2 + -1 \cdot 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

9.2 # 64

the determinant of $B = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$ is given by $\det(B) = 1 \cdot 1 - 1 \cdot 2 = -1$

B is invertible, and its inverse is $B^{-1} = -1 \cdot \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 2 & -1 \end{pmatrix}$

since B is invertible, then the solution to $BX = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is unique.

Namely, $X = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

9.3 #2

(a) $A(\vec{x}+\vec{y}) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix} = \begin{pmatrix} a_{11}(x_1+y_1) + a_{12}(x_2+y_2) \\ a_{21}(x_1+y_1) + a_{22}(x_2+y_2) \end{pmatrix}$

$$= \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix} + \begin{pmatrix} a_{11}y_1 + a_{12}y_2 \\ a_{21}y_1 + a_{22}y_2 \end{pmatrix}$$

$$= A\vec{x} + A\vec{y}$$

Thus, $A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y}$.

(b) $A(\lambda\vec{x}) = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} \lambda x_1 \\ \lambda x_2 \end{pmatrix} = \begin{pmatrix} a_{11}\lambda x_1 + a_{12}\lambda x_2 \\ a_{21}\lambda x_1 + a_{22}\lambda x_2 \end{pmatrix}$

$$= \lambda \begin{pmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{pmatrix}$$

$$= \lambda(A\vec{x})$$

Thus, $A(\lambda\vec{x}) = \lambda(A\vec{x})$.

Exercise 1

To show that B is the inverse of A, we want $AB = BA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$AB = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{ad-bc} \begin{pmatrix} ad-bc & -ab+ab \\ cd-cd & -bc+ad \end{pmatrix}$$

$$= \frac{1}{ad-bc} \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

similarly, we compute BA.

$$BA = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \frac{1}{ad-bc} \begin{pmatrix} da-bc & db-bd \\ -ca+ac & -cb+ad \end{pmatrix}$$

$$= \frac{1}{ad-bc} \begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

It follows that B is the inverse of A.